25/05/2020 **JESUS AND MARY SCHOOL AND COLLEGE** MODULE 4

**CLASS-10 (MATHS)**

**CHAPTER NAME – MATRICES**

**Note: Module-1 is referred to the Part-1 video uploaded for this chapter on 18th May, 2020**

TOPICS:

* To multiply a matrix by a number
* To multiply two given matrices.
* Properties of multiplication of matrices.
* Multiplication of a matrix by an Identity Matrix.
* How to find the matrix if it is missing in product of two matrices

In our Part – 1 video prepared for this chapter we learnt how to find the order of a matrix; to find the value of a defined element of a matrix and to add or subtract two or more matrices.

**MULTIPLICATION OF A MATRIX BY A NUMBER**

If *k* is any number and A is a matrix then the matrix *k*A is obtained by multiplying each element of the matrix A by the number *kI.*

Consider the following example.

If A = $\left[\begin{matrix}6&2\\0&5\end{matrix}\right]$ then find 3A.

* Step 1: First multiply 3 in 6 (3 × $6$ =$18$ )
* Step 2: Multiply 3 in 2 (3 × $2$ = $6 )$
* Step 3: Multiply 3 in 0 (3 × $0$ = $0 )$
* Step 4: Finally multiply 3 in 5 (3 × $5$ = $15 )$

So the result of 3A = $\left[\begin{matrix}18&6\\0&15\end{matrix}\right]$

**MULTIPLICATION OF MATRICES**

**IMPORTANT:** *We can only multiply two matrices if the number of columns in the first matrix is*

 *same as the number of rows in the second matrix.*

Example:

1. Multiplying a 2 x 3 matrix by a 3 x 4 matrix is possible and it gives a 2 x 4 matrix as the answer.
2. Multiplying a 7 x 1 matrix by a 1 x 2 matrix is okay; it gives a 7 x 2 matrix.
3. Multiplication of a 4 x 3 matrix with a 2 x 3 matrix is **NOT** possible.

**HOW TO MULTIPLY 2 MATRICES**

Let’s take an example of two matrices having alphabets as elements. We’ll see a numbers example afterwards.

**Given:** A =$\left[\begin{matrix}a&b\\c&d\end{matrix}\right]$; B =$\left[\begin{matrix}e&f\\g&h\end{matrix}\right]$, find AB.

**Procedure:**

1. We multiply and add the elements as follows. We work across the 1st row of the first matrix, multiplying down the 1st column of the second matrix, element by element. We add the resulting products. Our answer goes in position a11 (top left) of the product matrix.

$\left[\begin{matrix}a&b\\c&d\end{matrix}\right]\left[\begin{matrix}e&f\\g&h\end{matrix}\right]$= $\left[\begin{matrix} ae+bg&\\&\end{matrix}\right]$

1. We do a similar process for the 1st row of the first matrix and the 2nd column of the second matrix. The result is placed in position a 12 of the product matrix.

$\left[\begin{matrix}a&b\\c&d\end{matrix}\right]\left[\begin{matrix}e&f\\g&h\end{matrix}\right]$=$\left[\begin{matrix}&af+bh\\&\end{matrix}\right]$

1. Now for the 2nd row of the first matrix and the 1st column of the second matrix. The result is placed in position a 21of the product matrix.

$\left[\begin{matrix}a&b\\c&d\end{matrix}\right]\left[\begin{matrix}e&f\\g&h\end{matrix}\right]$= $\left[\begin{matrix}&\\ce+dg&\end{matrix}\right]$

1. Finally, we multiply the 2nd row of the first matrix and the 2nd column of the second matrix. The result is placed in position a 22 of the product matrix.

$\left[\begin{matrix}a&b\\c&d\end{matrix}\right]\left[\begin{matrix}e&f\\g&h\end{matrix}\right]$= $\left[\begin{matrix}&\\&cf+dh\end{matrix}\right]$

So the result of multiplying our 2 matrices is as follows:

$\left[\begin{matrix}a&b\\c&d\end{matrix}\right]\left[\begin{matrix}e&f\\g&h\end{matrix}\right]$= $\left[\begin{matrix}ae+bg&af+bh\\ce+dg&cf+dh\end{matrix}\right]$

Now let’s see a number example.

If B =$ \left[\begin{matrix}2&-2\\5&-5\end{matrix}\right]$and C =$\left[\begin{matrix}3&4\\5&6\end{matrix}\right]$, compute BC and CB.

BC =$ \left[\begin{matrix}2&-2\\5&-5\end{matrix}\right]\left[\begin{matrix}3&4\\5&6\end{matrix}\right]= \left[\begin{matrix}2 x 3+\left(-2 \right) x 5&2 x 4+(-2) x 6\\5 x 3+( -5) x 5&5 x 4+\left(-5 \right) x 6\end{matrix}\right] $= $\left[\begin{matrix}6-10 & 8-12\\15-25&20-30\end{matrix}\right]$

 = $\left[\begin{matrix} -4&-4\\-10&-10\end{matrix}\right]$

CB = $\left[\begin{matrix}3&4\\5&6\end{matrix}\right]\left[\begin{matrix}2&-2\\5&-5\end{matrix}\right]$ = $\left[\begin{matrix}3 x 2+4 x 5&3 x \left(-2\right)+ 4 x \left(-5\right)\\5 x 2+6 x 5&5 x \left(-2 \right) + 6 x \left(-5 \right)\end{matrix}\right]$ =$\left[\begin{matrix}6+20 &-6-20\\10+30&-10-30\end{matrix}\right]$

 =$ \left[\begin{matrix}26&-26\\40&-40\end{matrix}\right]$

In the above given manner we can multiply any 2 given matrices.

**PROPERTIES OF MULTIPLICATION OF MATRICES**

1. Multiplication of matrices is associative i.e. if A B and C are matrices conformable for multiplication, then

(AB) C = A (BC).

1. Multiplication of matrices is distributive with respect to addition i.e. if A, B and C are matrices conformable for the requisite addition and multiplication , then

A (B + C) = AB + AC and

(A + B) C = AC + BC

1. Cancellation law for the multiplication of matrices may not hold i.e.

if AB = AC then B= C may not be true, A $\ne $ 0.

1. Multiplication of matrices is not commutative i.e.

AB $\ne $BA

1. AB = 0, even though A$\ne $ 0 and B $\ne $0.

**MULTIPLICATION OF A MATRIX BY AN IDENTITY MATRIX**

First of all let’s see what is an identity matrix?

*“****Identity matrix is a square matrix in which all other elements are ‘zero’ except the elements on***

 ***principal diagonal”.***

It is denoted with I. e.g. $\left[\begin{matrix}1&0\\0&1\end{matrix}\right]$; $\left[\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}\right]$

**Note:** *Principal diagonal starts from first element of first row to last element of last row. In identity*

 *matrix* $\left[\begin{matrix}1&0\\0&1\end{matrix}\right]$(1 1) *is principal diagonal and in identity matrix* $\left[\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}\right] $(1 1 1) *is principal*

 *diagonal.*

**Example 1:** If A =$\left[\begin{matrix}3&4\\5&6\end{matrix}\right]$; I =$\left[\begin{matrix}1&0\\0&1\end{matrix}\right]$, find AI

**Solution:** AI = $\left[\begin{matrix}3&4\\5&6\end{matrix}\right]\left[\begin{matrix}1&0\\0&1\end{matrix}\right]$=$\left[\begin{matrix}3 x 1+4 x 0&3 x 0+4 x 1 \\5 x 1+6 x 0&5x0+6 x 1 \end{matrix}\right]$ = $\left[\begin{matrix}3+0&0+ 4\\5+0 &0+ 6\end{matrix}\right]$ = $\left[\begin{matrix}3&4\\5&6\end{matrix}\right]$

**Example 2:** If A=$\left[\begin{matrix}2&7\\5&8\end{matrix}\right],$ find A2 + 2I

**Solution:** Here I will be a 2 x 2 matrix because A is a 2 x 2 matrix.

 First we will find **A2** that means **A x A**

 A2 = $\left[\begin{matrix}2&7\\5&8\end{matrix}\right]\left[\begin{matrix}2&7\\5&8\end{matrix}\right]$= $\left[\begin{matrix}2 x 2+7 x 5 &2 x 7+7 x 8\\5 x 2+8 x 5 &5 x 7+ 8 x 8\end{matrix}\right]$ = $\left[\begin{matrix}4+35&14+56\\10+40 &35+64\end{matrix}\right]$

 = $\left[\begin{matrix}39&70\\50&99\end{matrix}\right]$

 A2 + 2I = $\left[\begin{matrix}39&70\\50&99\end{matrix}\right]$ + 2$\left[\begin{matrix}1&0\\0&1\end{matrix}\right]$ = $\left[\begin{matrix}39&70\\50&99\end{matrix}\right]+ \left[\begin{matrix}2&0\\0&2\end{matrix}\right]$

 = $\left[\begin{matrix}41&70\\0&101\end{matrix}\right]$

**HOW TO FIND THE MATRIX IF IT IS MISSING IN PRODUCT OF TWO MATRICES**

Let’s consider the following example.

**Example 3:** Given $\left[\begin{matrix} 2&1\\-3&4\end{matrix}\right] X$ =$\left[\begin{matrix} 7\\ 6\end{matrix}\right]$ , write

1. the order of the matrix X ii) the matrix X

**Solution:**

Step 1: First we will find the order of missing matrix, here in this case order of matrix X. to do that

 we will apply the following logic;

$A\_{m×n}×B\_{n×p}=C\_{m×p}$

 Applying the above logic we can very easily find out the order of matrix X that will be$ 2×1$.

Step 2: Now let a $2×1 $matrix i.e. $\left[\begin{matrix} a\\ b\end{matrix}\right]$

 Now our question is like –

 $\left[\begin{matrix} 2&1\\-3&4\end{matrix}\right]\left[\begin{matrix} a\\ b\end{matrix}\right]$= $\left[\begin{matrix} 7\\ 6\end{matrix}\right]$

**WORKSHEET-2**

**Q.1-** If A = $\left[ \begin{matrix}3& 5\\4&-2\end{matrix} \right]$ and B =$\left[ \begin{matrix}2\\4\end{matrix} \right]$, is the product AB possible? Give a reason. If yes, find AB.

**Q.2-** If A =$\left[ \begin{matrix}3&7\\2&4\end{matrix} \right]$, B = $\left[ \begin{matrix}0&2\\5&3\end{matrix} \right]$ and C =$\left[ \begin{matrix} 1&-5\\-4& 6\end{matrix} \right]$, find$ AB-5C$.

**Q.3-** If A = $\left[\begin{matrix}-1&3\\ 2&4\end{matrix} \right]$ and B =$\left[\begin{matrix} 2&-3\\-4&-6\end{matrix} \right]$, find the matrix $AB+BA$.

**Q.4-** If A =$\left[ \begin{matrix}1&2\\2&3\end{matrix} \right]$, B = $\left[ \begin{matrix}2&1\\3&2\end{matrix} \right]$ and C = $\left[ \begin{matrix}1&3\\3&1\end{matrix} \right]$, find $C(B-A)$.

**Q.5-** If A =$\left[ \begin{matrix}2& 1\\0&-2\end{matrix} \right]$, B = $\left[\begin{matrix} 4& 1\\-3&-2\end{matrix} \right]$ and C =$\left[ \begin{matrix}-3&2\\-1&4\end{matrix} \right]$, find $A^{2}+AC-5B$.

**Q.6-** If $X=\left[\begin{matrix} 4&1\\-1&2\end{matrix}\right]$, show that $6X-X^{2}=9I$ where I is the unit matrix.

**Q.7-** Show that $\left[ \begin{matrix}1&2\\2&1\end{matrix} \right]$ is a solution of the matrix equation $X^{2}-2X-3I=O$ where I is the unit

 matrix of order 2.

**Q.8-** Find the value of *x* and *y* if $\left[\begin{matrix}x+y&y\\y&x-y\end{matrix}\right]\left[\begin{matrix} 2\\-1\end{matrix}\right]=\left[\begin{matrix}3\\2\end{matrix}\right].$

**Q.9-** If $A=\left[\begin{matrix}2&3\\1&2\end{matrix}\right]$, find *x, y* so that $A^{2}=xA+yI$.

**Q.10-** If $M×\left[\begin{matrix}1&1\\0&2\end{matrix}\right]=\left[\begin{matrix}1&2\end{matrix}\right]$ where M is a matrix.

1. State the order of matrix M.
2. Find the matrix M.

**Note**- **Please do this assignment in your copies. It will be checked when the school re-opens.**

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